I Semester B.A./B.Sc. Examination, Nov./Dec. 2013
(Semester Scheme) (2011-12 and Onwards) (NS)
MATHEMATICS – I

Time : 3 Hours

Max. Marks : 100

Instruction : *Answer* ***all*** *questions.*

I. Answer **any fifteen** of the following questions : (15×2=30)

1) Reduce the matrix $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$ to echelon form.

2) For what value of x is the matrix $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$ singular ?

3) Find the value of K such that the following system of equations has non-trivial solutions.

$(K-1)x + (3K+1)y + 2Kz = 0,\ (K-1)x + (4K-2)y + (K+3)z = 0$

$2x + (3K+1)y + 3(K-1)z = 0.$

4) Find the eigen values of the matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

5) Find the n^{th} derivative of $e^{3x}\sin 4x$.

6) Find the n^{th} derivative of $\cos 2x \sin 3x$.

7) If $u = e^{ax}\sin by$ find $\frac{\partial^2 u}{\partial x \partial y}$.

8) If $u = \log\left(\frac{x^{10} + y^{10}}{x + y}\right)$ find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

P.T.O.

9) If $u = \frac{x}{y} + \frac{y}{x}$ where $x = t$, $y = t + 1$. Find $\frac{du}{dt}$.

10) If $x = u(1 - v)$, $y = uv$ find $\frac{\partial(x, y)}{\partial(u, v)}$.

11) Evaluate $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right) dx$.

12) Evaluate $\int_0^{a} (a^2 + x^2)^{3/2} dx$.

13) Find the ratio in which the line joining the points (2, 4, 5) and (3, 5, –4) is divided by xy – plane.

14) Find the locus of a point which is equidistant from the points (1, 2, 3) and (3, 2, –1).

15) Define direction cosines and direction ratios of a line.

16) Find the equation of the line passing through the point (3, 4, 5) and is parallel to the vector $2i + 2j - 3k$.

17) Find the angle between the line $\frac{x+1}{3} = \frac{y}{1} = \frac{z-4}{2}$ and the plane $x + y + z = 6$.

18) Find K so that the lines $\frac{x-1}{2} = \frac{y-2}{2K} = \frac{z+1}{-1}$ and $\frac{x+1}{K} = \frac{y+1}{4} = \frac{z-2}{1}$ are perpendicular to each other.

19) Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$.

20) If a right circular cone has three mutually perpendicular generations, show that the semi vertical angle is $\tan^{-1}\sqrt{2}$.

II. Answer **any two** questions. **(2×5=10)**

1) Find the rank of the matrix $\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 10 \end{bmatrix}$.

2). Show that the equations $x + y + z = 6$, $x + 2y + 3z = 14$, $x + 4y + 7z = 30$ are consistent and solve them.

3) Verify the Cayley-Hamilton Theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$.

4) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$.

(4×5=20)

III. Answer **any four** questions.

1) Find the n^{th} derivative of sinx. sin2x. sin3x.

2) If $y = (\sinh^{-1} x)^2$ show that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2 y_n = 0$.

3) State and prove the Euler's theorem for homogeneous functions in x and y of degree 'n'.

4) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1-4\sin^2 u)\sin 2u.$$

5) If $u = f(2x-3y, 3y-4z, 4z-2x)$. Show that $6\frac{\partial u}{\partial x} + 4\frac{\partial u}{\partial y} + 3\frac{\partial u}{\partial z} = 0$.

6) If $u = 2xy$ and $v = x^2 - y^2$ and $x = r\cos\theta$, $y = r\sin\theta$, prove that

$$\frac{\partial(u, v)}{\partial(r, \theta)} = -4r^3.$$

(2×5=10)

IV. Answer **any two** questions.

1) Evaluate $\int_0^{\pi} x \sin^4 n \cos^6 x \, dx$.

2) Find the reduction formula for $\int \sin^n x \, dx$ for n being a positive integer.

3) By applying the Leibnitz rule of differentiation under integral sign, evaluate

$$\int_0^1 \frac{x^{\alpha}-1}{\log x} dx.$$

V. Answer **any four** questions. **(4×5=20)**

1) Find the direction cosines of the two lines whose direction cosines are connected by the relations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$.

2) Find the equation of the plane which makes the intercepts a, b, c on the coordinate axes.

3) Show that the points (0, –1, –1), (4, 5, 1), (3, 9, 4) and (–4, 4, 4) lie on a plane. Find the equation of the plane.

4) Find the equation of the plane passing through two points A(2, 2, 1) and B(9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$.

5) Show that the lines $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{1}$ and $\frac{x-2}{3} = \frac{y-2}{2} = \frac{z-6}{4}$ are coplanar and find the equation of the plane containing them.

6) Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-6}{4} = \frac{z-5}{5}$.

VI. Answer **any two** questions. **(2×5=10)**

1) Prove that the condition for two spheres $x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ and $x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$ to cut orthogonally is $2u_1u_2 + 2v_1v_2 + 2w_1w_2 = d_1 + d_2$.

2) Find the equation of the right circular cone whose vertex is (2, –3, 5) axis makes equal angles with the coordinate axes and with semi-vertical angle $\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$.

3) Find the equation of the right circular cylinder of radius 2 and whose axis is $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-3}{5}$.

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I Semester B.A./B.Sc. Examination, Nov./Dec. 2013
(Semester Scheme)
(OS) (Prior to 2011-12)
MATHEMATICS – I

Time : 3 Hours Max. Marks : 90

Instruction : *Answer **all** questions.*

I. Answer **any fifteen** of the following questions. (15×2=30)

1) If S = {1, 2, 3, 4, 5, 6} is the replacement set, find the truth set of the proposition p(x) : $x + 2 < 7$.

2) Write the negation of the statement 'all integers are rational and some integers are even'.

3) Find the equivalence relation induced by the partition {{p}, {q}, {r}, {s}} on the set A = {p, q, r, s}.

4) Define one-one function.

5) Find the nth derivative of cos 3x.

6) Find the nth derivative of $e^{2x}\sin x$.

7) If $u = xy + yz + zx$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 2(x + y + z)$.

8) If $f(x, y) = 2xy$, $x = \cos t$ and $y = \sin t$ find $\frac{df}{dt}$.

9) Find $\frac{dy}{dx}$ using partial derivatives for $x^2 + y^2 = a^2$.

10) If $x = u(1 + v)$, $y = v(1 + u)$ prove that $\frac{\partial(x, y)}{\partial(u, v)} = 1 + u + v$.

11) Evaluate $\int_0^{\pi/2} \sin^{-1} x \, dx$.

12) Evaluate $\int_0^{\pi} x \sin^3 x dx$.

13) Show that the points (1, 2, 3), (3, 7, 7) and (5, 12, 11) are collinear.

14) Find the equation of the line in vector and Carterian forms whose position vector is $\vec{a} = (2, 1, -3)$ and parallel to the vector $\vec{v} = \hat{i} - 2\hat{j} + 3\hat{k}$.

15) Show that the line $\frac{x+5}{3} = \frac{y-1}{2} = \frac{z-1}{-1}$ is parallel to the plane $2x - 5y - 4z + 9 = 0$.

16) Find the equation of the plane passing through the point (2, –4, 5) and is parallel to the plane $4x + 2y - 7z + 6 = 0$.

17) Find the angle between the line joining the points (1, 2, 3) and (4, 5, 7) and the plane $x + 3y - 3z = 4$.

18) Find the equation of the sphere whose centre is (1, –1, 1) and the radius is 5.

19) Find the equation of the right circular cylinder whose radius is 2 units and axis passes through the points (1, –3, 2) and (–1, –2, –3).

20) Find the equation of a right circular cone with vertex (1, –1, 0), the axis $\frac{x-1}{1} = \frac{y+1}{3} = \frac{z-0}{2}$ and semi vertical angle is 60°.

(2×5=10)

II. Answer **any two** of the following questions.

1) With usual notations prove that $T[p(x) \wedge q(x)] = T[p(x)] \cap T[q(x)]$

2) Give a direct proof of the statement "the sum of two integers each of which is divisible by n is divisible by n".

3) If $f : A \to B$ and $g : B \to C$ are two bijective functions then prove that gof is also bijective function.

4) If $f : R \to R$ and $g : R \to R$ defined by $f(x) = x^2 + 3$, $g(x) = 3x + 1$ find gof and fog.

(3×5=15)

III. Answer **any three** of the following questions.

1) State and prove Leibnitz's theorem.

2) If $y = e^{\sin^{-1} x}$, prove that $(1 - x^2) y_{n+2} - (2n + 1) xy_{n+1} - (n^2 + 1) y_n = 0$.

3) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$.

4) If $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$, $x = 2t + 1$, $y = t + 1$, $z = t$ find $\frac{df}{dt}$.

5) If $u = xyz$, $v = xy + yz + zx$ and $w = x + y + z$ then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

IV. Answer **any two** of the following questions. **(2×5=10)**

1) Find reduction formula for $\int \sin^n x\, dx$ where n is a +ve integer.

2) Evaluate $\int_0^\infty \frac{x^2}{(1+x^2)^3} dx$.

3) Using Leibnitz's rule of differentiation under integral sign evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x} dx$ where $\alpha > 0$ is a parameter.

V. Answer **any three** of the following questions. **(3×5=15)**

1) Find the angle between the two lines whose direction cosines are given by the equations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$.

2) Find the value of 'a' given that the points (a, 2, 0), (1, 1, 0), (1, 2, 1) and (–2, 2, –1) are coplanar.

3) Find the vector and Carterian form of a straight line passing through two given points.

4) Find the equation of the plane which passes through the point (–1, 3, 2) and is perpendicular to each of the planes $x + 2y + 2z = 5$ and $3x + 3y + 2z = 0$.

5) Prove that the lines $\frac{x-3}{2} = \frac{y-2}{-5} = \frac{z-1}{3}$ and $\frac{x-1}{-4} = y+2 = \frac{z-6}{2}$ are coplanar and find the equation to the plane containing them.

VI. Answer **any two** of the following questions. **(2×5=10)**

1) Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) and (2, –1, 1).

2) Find the equation of the right circular cone generated by revolving the line $\frac{x-1}{-1}=\frac{y-2}{3}=\frac{z-3}{3}$ about the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ as its axis.

3) Find the equation of the right circular cylinder whose generators touch the sphere $x^2+y^2+z^2=9$ and are parallel to the line $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z-3}{5}$.